Let’s start this post with a question: In how many different ways (relative to each other) can 5 friends sit around a round table if all the seats are identical?

I guess that most of you will be able to answer it –> 4! = 24 ways

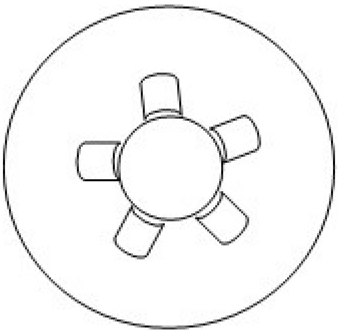
After all, you know the formula of circular arrangement which is (n-1)! But, many of you probably do not understand exactly why the formula is (n-1)! Today’s post is focused on explaining the concept of circular arrangement. If you are wondering why you need to know the theory behind the formula when all you need to do in a question is apply the formula and get the answer, here is why — you will be able to solve a straight forward 500-600 level question knowing just the formula but you will not get the 700+ level GMAT question correct. You need to understand the basics behind the formula so that you can apply it with modifications in more inventive situations. I will give you a couple of questions after discussing the theory and you will see what I mean. Right now, let’s focus on the question posed above.

Question 6: In how many different ways (relative to each other) can 5 friends sit around a round table if all the seats are identical?

Now there are two ways to explain the formula used here. I will give both. See what makes more sense to you.

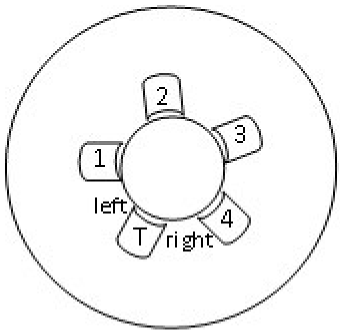
Method 1:

When we pose questions on circular arrangement, the different arrangements we are looking for are those in which people are sitting differently relative to each other. So ignore any other point of reference. Say there is a monochromatic circular room with a circular door right in the middle of the roof and people enter the room Mission-Impossible style. There is a circular table with 5 chairs around it all placed at equal distances. When the first person, Mr. T, drops in, which chair should he sit on? Can we say that it is immaterial which chair he sits on since all the seating spaces are exactly the same?



Since there is no one else sitting as yet, for him every seat is the same. In how many ways can he choose a seat then? In only one way (since every way in which he chooses a seat is the same). No matter which chair he sits on, the arrangement remains exactly the same.

Now when the second person, Mr. B, drops in, in how many different ways can he occupy a seat? Mr. B has four choices. Each one of the seats is different relative to where Mr. T. sits. Mr. B can choose to sit on the left of Mr. T, next to him — i.e., on seat number 1. Or he can choose to sit on the left of Mr. T but with a seat between them i.e. seat number 2. Or he can sit on the right of Mr. T, next to him i.e. seat number 4. Or he can sit on the right of Mr. T but with a seat between them i.e. seat number 3. Each one of the 4 seats are different relative to Mr. T.



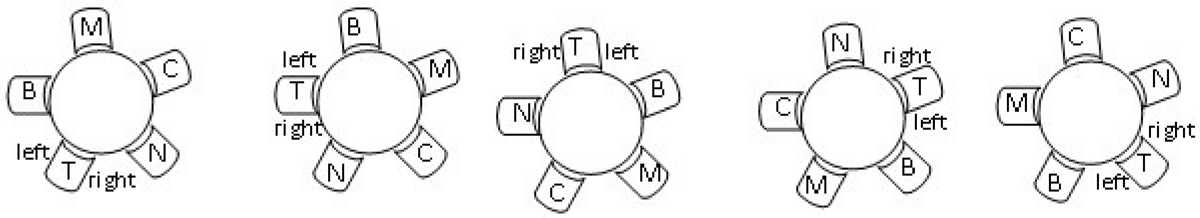
So Mr. B can choose a seat in 4 ways.

Next, when Mr. M drops in, he can choose a seat in 3 ways and so on till the last person has 1 seat left for him. The total number of arrangements then are 4*3*2*1 = 24 (think of your basic counting principle here). For the first person, all seats are the same so he can choose in 1 way. He creates a frame of reference and thereafter, every seat is distinct (relative to him). So the rest of the (n-1) people can sit in (n-1) seats in (n-1)! ways (using Basic Counting Principle).

This is how we arrive at the formula (n-1)!

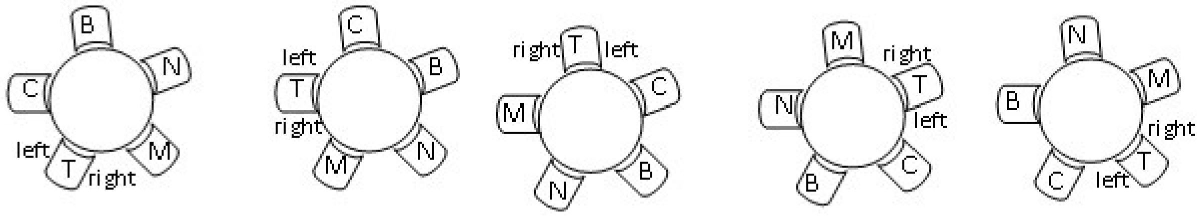
Method 2:

Let’s say we have arranged the 5 people on the 5 seats. Would you say that the following 5 combinations are exactly the same (relative to people)?



Mr. B is next to Mr. T on the left and Mr. N is next to Mr. T on the right. Mr. M is to the left of Mr. B and Mr. C is to the right of Mr. N.

Similarly, these 5 arrangements are exactly the same too (but different from the arrangements above).



Mr. C is next to Mr. T on the left and Mr. M is next to Mr. T on the right. Mr. B is to the left of Mr. C and Mr. N is to the right of Mr. M.

Every group of 5 arrangements is actually a single arrangement since relative to one another, people are arranged in the same way. So we divide 5! by 5 to count only the actual distinct arrangements.

Hence we get the formula n!/n = (n-1)!

I like to think in terms of method 1 since it helps me take care of a lot of variations on simple circular arrangement questions. Let's look at one of these variations.

Question 7: There are 5 people – A, B, C, D and E. They have to sit around a circular table with 5 chairs such that A can sit neither next to D nor next to E. How many such distinct arrangements are possible?

Solution: A can sit neither next to D nor next to E. So A has to sit next to B and C. Let's say we first make A sit on any one chair. In how many ways can A choose his chair? In only 1 way because all the chairs are the same for him (he is the first person being seated). Now B and C have to sit next to him. B can sit on the right of A and C can sit on the left of A OR B can sit on the left of A and C can sit on the right of A. There are two ways in which you can arrange B and C around A. Now there are 2 chairs left and two people D and E. D can choose his chair in 2 ways since the two seats are distinct (relative to A, B and C) and the last chair will be left for E.

Total number of arrangements = 1*2*2*1 = 4 ways

Note: In case nothing is mentioned, in a circular arrangement, two seating arrangements are considered different only when the positions of the people are different relative to each other. If it is given that the seats are distinct (say they are different colored), then the number of arrangements is n! (same as in the case of linear arrangements)

Now, let me leave you with a question which is based on the concept of circular arrangement and can be easily solved if you understand the theory above. I will discuss its solution in the next post.

Question 7: In how many different ways (relative to each other) can 8 friends sit around a square table with 2 seats on each side of the table?